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Technical Note

A Quantitative Analysis of the Feynman- and Rossi-Alpha Formulas with Multiple Emission Sources

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Abstract—*Recently, analytical formulas have been derived for the Feynman- and Rossi-Alpha measurements in accelerator-driven systems. In such systems, due to the multiplicity of the sources, the Feynman- and Rossi-Alpha formulas contain additional terms as compared with the traditional cases. A numerical evaluation of these formulas for systems with such sources is given. An assessment of the contribution of the terms that are novel as compared to the traditional formula is made. These include the terms arising from the source multiplicity, and the prompt-delayed and delayed-delayed correlations. Further, the consequences of averaging the delayed-neutron families are analyzed. Finally, a comparison is made, assuming traditional core material and one possible type of future accelerator-driven system.*

I. INTRODUCTION

Formulas for fluctuation-based measurement of reactor parameters, notably the negative reactivity in subcritical systems with a source, have been known for a long time.[1–5] The two most commonly used ones are the Feynman- and Rossi-alpha formulas, respectively. These formulas have been analyzed quantitatively in the past, as shown, for example, in Refs. 2 and 3, as functions of the subcritical reactivity.

Recently,[6–12] the Feynman- and Rossi-alpha formulas have been rederived for so-called accelerator-driven subcritical (ADS) systems. The need for the rederivation arose because such future systems will use a spallation source,[13–16] which has statistical properties different from those in traditional systems. In the latter, a simple radioactive source is used, which emits one neutron at a time and has, thus, Poisson statistics. The spallation source that will be used in ADS systems emits several neutrons simultaneously (or nearly simultaneously), which are all correlated, in each spallation event. Such a source has so-called compound Poisson statistics.[17]

The most detailed derivation of the Feynman- and Rossi-alpha formulas was given recently by Kuang and Pázsit.[11] In that work, a multiple source as well as six delayed-neutron groups were assumed. Further, unlike in all work in traditional systems, the correlations between prompt and delayed neutrons, as well as those between two delayed neutrons in the fission source term, were accounted for explicitly.

This technical note gives a quantitative evaluation of the Feynman- and Rossi-alpha formulas given in Ref. 11. The differences between the traditional formula (assuming a Poisson source) and the new formula (spallation source) are investigated quantitatively for different degrees of subcritical reactivity. The relative contribution of the source correlation term as well as that of the prompt-delayed and delayed-delayed fission terms are assessed. These terms were neglected in all previous work already in the starting master equation. Here, the assessment of their negligible contribution to the result is made in the explicit solution. The differences between using six delayed-neutron groups explicitly and one averaged delayed-neutron group are also assessed. Finally, the formulas are evaluated for traditional cores using ^{235}U

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as the fissile material and for a hypothetical ADS core containing ^{233}U as the fissile material.

II. THE FEYNMAN- AND ROSSI-ALPHA FORMULAS

II.A. The Feynman-Alpha Formula

The Feynman-alpha formula is usually written in terms of the function $Y(t)$, equal to the ratio of the modified variance of the detector counts to the mean value of detector counts, i.e.,

$$Y(t) \equiv \frac{\tilde{\sigma}_{ZZ}^2}{\tilde{Z}} - 1 = \frac{\tilde{\mu}_{ZZ}(t)}{\tilde{Z}(t)} \quad . \tag{1}$$

Here, $\tilde{Z}(t)$ is the expected value of the detector counts in a stationary subcritical system between time 0 and t, and $\tilde{\sigma}_{ZZ}^2$ is its variance. In Ref. 11, the following expression was derived for the $Y(t)$ function of the Feynman-alpha formula:

$$\begin{aligned} Y(t) = {} & 2\epsilon\lambda_f^2\left[\langle \nu_p(\nu_p - 1)\rangle + \langle q(q-1)\rangle \frac{\nu}{\bar{q}}(-\rho)\right] \\ & \times \sum_{i=0}^{6} \omega_i f_i(t) \\ & - 2\epsilon\lambda_f^2 \sum_{j=1}^{6} \lambda_j^2 (2\langle \nu_p \nu_{d_j}\rangle + \langle \nu_{d_j}(\nu_{d_j} - 1)\rangle) \\ & \times \sum_{i=0}^{6} \frac{\omega_i f_i(t)}{s_i^2 - \lambda_j^2} - 4\epsilon\lambda_f^2 \sum_{\substack{i,j=1 \\ i<j}}^{6} \frac{\lambda_i \lambda_j}{\lambda_i + \lambda_j} \langle \nu_{d_i} \nu_{d_j}\rangle \\ & \times \left(\lambda_i \sum_{k=0}^{6} \frac{\omega_k f_k(t)}{s_k^2 - \lambda_i^2} + \lambda_j \sum_{k=0}^{6} \frac{\omega_k f_k(t)}{s_k^2 - \lambda_j^2}\right) \quad . \end{aligned} \tag{2}$$

Here the following notations were used:

$$\omega_i \equiv \frac{z_i}{s_i} \sum_{j=0}^{6} \frac{z_j}{s_i + s_j} \quad , \quad i = 0,1,\ldots,6 \quad , \tag{3}$$

$$f_i(t) \equiv \left(1 + \frac{1 - e^{s_i t}}{s_i t}\right) = \left(1 - \frac{1 - e^{-\alpha_i t}}{\alpha_i t}\right) \quad , \qquad (\alpha_i = -s_i), i = 0,1,\ldots,6 \tag{4}$$

and

$$z_k = \frac{\prod_{j=1}^{6}(s_k + \lambda_j)}{\prod_{j \neq k}(s_k - s_j)} = 1 \Bigg/ \left(1 + \frac{1}{\Lambda} \sum_{j=1}^{6} \frac{\beta_j \lambda_j}{(s_k + \lambda_j)^2}\right) \quad , \qquad k = 0,1,\ldots,6 \quad . \tag{5}$$

The s_k, $k = 0,1,\ldots,6$, are the solutions of the characteristic equation (the inhour equation):

$$s + \alpha - \frac{1}{\Lambda} \sum_{j=1}^{6} \frac{\beta_j \lambda_j}{s + \lambda_j} = 0 \quad , \tag{6}$$

where

$$\alpha \equiv -\frac{\rho - \beta}{\Lambda} > 0 \quad . \tag{7}$$

As is known and as will also be seen shortly (see Table I), for all practical cases,

$$\alpha \approx \alpha_0 \quad , \tag{8}$$

where the α_i were defined by Eq. (4). The rest of the notations are standard. A nomenclature of all notations used in this paper is given in Ref. 11.

As will be seen shortly, the $|s_k|$ is so close to λ_k for $k = 1,\ldots,6$, that Eq. (2) is not practical to use for a numerical evaluation. However, this apparent singularity can be eliminated. To see this, we introduce

$$\begin{aligned} u(k,i) \equiv {} & \frac{\omega_k \lambda_i^2}{s_k^2 - \lambda_i^2} = \frac{\lambda_i^2}{s_k^2 - \lambda_i^2} \frac{z_k}{s_k} \sum_{j=0}^{6} \frac{z_j}{s_k + s_j} \\ = {} & \frac{\lambda_i^2}{s_k} \frac{1}{s_k^2 - \lambda_i^2} \frac{\prod_{j=1}^{6}(s_k + \lambda_j)}{\prod_{j \neq k}(s_k - s_j)} \sum_{j=0}^{6} \frac{z_j}{s_k + s_j} \\ = {} & \frac{\lambda_i^2}{s_k} \frac{1}{s_k - \lambda_i} \frac{\prod_{j \neq i}(s_k + \lambda_j)}{\prod_{j \neq k}(s_k - s_j)} \sum_{j=0}^{6} \frac{z_j}{s_k + s_j} \quad . \end{aligned} \tag{9}$$

TABLE I

Roots of the Characteristic Equations for $\Lambda = 5 \times 10^{-5}$ s; $\beta = 0.0065$*

k_{eff}	α	$-s_0$	$-s_1$	$-s_2$	$-s_3$	$-s_4$	$-s_5$	$-s_6$
0.99	332.02	332.179	0.01083	0.02792	0.08801	0.19243	1.11294	2.89127
0.95	1182.6	1182.68	0.00618	0.02933	0.10970	0.19967	1.03346	3.00267
0.8	5130	5130.01	0.01238	0.02594	0.10089	0.20449	1.13342	2.98075

*The roots $a_i = -s_i$ are given in units of s^{-1}.

With this, Eq. (2) can be rewritten as

$$Y(t) = 2\epsilon\lambda_f^2 \sum_{k=0}^{6} D_k f_k(t) \quad , \tag{10}$$

where

$$\begin{aligned} D_k = {} & \left[\langle \nu_p(\nu_p - 1)\rangle + \langle q(q-1)\rangle \frac{\nu}{\bar{q}}(-\rho)\right]\omega_k \\ & - \sum_{j=1}^{6} (2\langle \nu_p \nu_{d_j}\rangle + \langle \nu_{d_j}(\nu_{d_j} - 1)\rangle) u(k,j) \\ & - 2 \sum_{\substack{i,j=1 \\ i<j}}^{6} \frac{1}{\lambda_i + \lambda_j} \langle \nu_{d_i}\nu_{d_j}\rangle [\lambda_j u(k,i) + \lambda_i u(k,j)] \quad . \end{aligned} \tag{11}$$

Since $s_i < 0$ for $i = 1 \ldots 6$, the $u(k,i)$ are free of any singularities. Thus, Eqs. (10) and (11) are fully regular expressions, suitable for numerical evaluation.

II.B. Rossi-Alpha Formula

The Rossi formula is usually written in the form

$$P_{rossi}(\tau)dt = \frac{\tilde{C}_{ZZ}(\tau)}{\tilde{Z}} \quad , \tag{12}$$

where $\tilde{C}_{ZZ}(\tau)$ is the stationary value of the covariance of the detector counts in infinitesimal time intervals (t, dt) and $(t+\tau, dt)$, and $\tilde{Z}$ is the first moment of the stationary detector counts in (t, dt). Because of stationarity, the quantities in Eq. (12) depend only on τ but not on t.

By redenoting τ as t, the formula derived by us[11] earlier is as follows:

$$\begin{aligned} & P_{rossi}(t)dt \\ & = \epsilon\lambda_f^2 dt \Bigg\{ \left[\langle \nu_p(\nu_p - 1)\rangle + \langle q(q-1)\rangle \frac{\nu}{\bar{q}}(-\rho)\right] \\ & \qquad \times \sum_{i=0}^{6} \omega_i g_i(t) \\ & \qquad - \sum_{j=1}^{6} \lambda_j^2 (2\langle \nu_p \nu_{d_j}\rangle + \langle \nu_{d_j}(\nu_{d_j} - 1)\rangle) \\ & \qquad \times \sum_{i=0}^{6} \frac{\omega_i g_i(t)}{s_i^2 - \lambda_j^2} - 2 \sum_{\substack{i,j=1 \\ i<j}}^{6} \frac{\lambda_i \lambda_j}{\lambda_i + \lambda_j} \langle \nu_{d_i}\nu_{d_j}\rangle \\ & \qquad \times \left(\lambda_i \sum_{k=0}^{6} \frac{\omega_k g_k(t)}{s_k^2 - \lambda_i^2} + \lambda_j \sum_{k=0}^{6} \frac{\omega_k g_k(t)}{s_k^2 - \lambda_j^2}\right)\Bigg\} \\ & = \epsilon\lambda_f^2 dt \sum_{k=0}^{6} D_k g_k(t) \quad . \end{aligned} \tag{13}$$

In this equation, all notations are the same as with the Feynman-alpha formula. The difference is that the functions $f_i(t)$ are replaced by the functions $g_i(t)$ that are defined as

$$g_i(t) \equiv -s_i e^{s_i t} = \alpha_i e^{-\alpha_i t} \quad ; \quad i = 0,1,\ldots,6 \quad . \tag{14}$$

III. THE SOLUTION OF THE CHARACTERISTIC EQUATION

First we shall consider a traditional thermal system; i.e., the core is assumed to contain ^{235}U as fissile material, which is of course only valid in conventional reactor systems. The data used for our calculation are taken from Ref. 18 and are listed in Table II.

Let

$$h(s) \equiv s + \alpha - \frac{1}{\Lambda} \sum_{j=1}^{6} \frac{\beta_j \lambda_j}{s + \lambda_j} \quad . \tag{15}$$

It is easy to see that $h(s)$ has six poles at $s = -\lambda_j, j = 1, \ldots, 6$. Further, it has the following properties:

1. $h(s)$ is a monotonic continuous function in any of the seven intervals $(-\infty - \lambda_6)$, $(-\lambda_6, -\lambda_5)$, $(-\lambda_5, -\lambda_4)$, $(-\lambda_4, -\lambda_3)$, $(-\lambda_3, -\lambda_2)$, $(-\lambda_2, -\lambda_1)$, and $(-\lambda_1, +\infty)$.

2. $\lim_{s \to -\lambda_j^+} h(s) = -\infty$; $\lim_{s \to -\lambda_j^-} h(s) = +\infty$; $\lim_{s \to -\infty} h(s) = -\infty$; and $h(0) = -(\rho/\Lambda)$.

The foregoing indicates that for $\rho < 0$, the characteristic equation, Eq. (6), has seven roots s_k, $k = 0,1,\ldots,6$, such that

$$\begin{aligned} & s_0 < -\lambda_6 < s_6 < -\lambda_5 < s_5 < -\lambda_4 < s_4 < -\lambda_3 \\ & \quad < s_3 < -\lambda_2 < s_2 < -\lambda_1 < s_1 < 0 \quad . \end{aligned}$$

If $\alpha \gg \lambda_6$, which is usually the case (compare Tables I and II), then

$$s_0 = -\alpha + \frac{1}{\Lambda} \sum_{j=1}^{6} \frac{\beta_j \lambda_j}{s_0 + \lambda_j} < -\alpha \ll -\lambda_6 \quad , \tag{16}$$

and the following approximation can be made to obtain s_0. That is,

$$s_0 = -\alpha + \frac{1}{\Lambda s_0} \sum_{j=1}^{6} \beta_j \lambda_j \tag{17}$$

or

$$s_0^2 + \alpha s_0 - \frac{1}{\Lambda} \sum_{j=1}^{6} \beta_j \lambda_j = 0 \quad ; \tag{18}$$

hence,

$$s_0 = \frac{1}{2}\left(-\alpha - \sqrt{\alpha^2 + \frac{4}{\Lambda} \sum_{j=1}^{6} \beta_j \lambda_j}\right) \quad . \tag{19}$$

TABLE II

Characteristics of Delayed Groups for Thermal Fission of ^{235}U; $\beta = 0.0065$; $\nu = 2.43$; $\langle \nu(\nu-1) \rangle = 4.7109$

	Delayed Group					
	1	2	3	4	5	6
Decay constant λ_i (s^{-1})	0.0124	0.0305	0.111	0.301	1.14	3.01
Relative abundance β_i/β	0.033	0.219	0.196	0.395	0.115	0.042

The other roots are calculated easily by use of the bisection method. For the data in Table II, one obtains the results shown in Table I.

IV. EFFECT OF DIFFERENT SOURCES

Now, to evaluate expression (11) for the functions D_k numerically, we need the values of the prompt-delayed and delayed-delayed second moments, respectively. These are not known, and thus following common practice (see, for example, Ref. 19), we shall assume

$$\langle \nu_p \nu_{d_j} \rangle = (1-\beta)\beta_j \ , \tag{20}$$

$$\langle \nu_{d_j}(\nu_{d_j}-1) \rangle = (\beta_j - 1)\beta_j \ , \tag{21}$$

and

$$\langle \nu_{d_i} \nu_{d_j} \rangle = \beta_i \beta_j \ . \tag{22}$$

With these assumptions, Eq. (11) can be rewritten as

$$\begin{aligned} D_k = {} & \{D_\nu[\nu(1-\beta)]^2 + D_q \nu \bar{q}(-\rho)\}\omega_k \\ & - \beta^2 \sum_{j=1}^{6} \left(2\,\frac{\nu(1-\beta)\beta_j}{\beta^2} + \frac{(\beta_j - 1)\beta_j}{\beta^2} \right) u(k,j) \\ & - 2\beta^2 \sum_{\substack{i,j=1 \\ i<j}}^{6} \frac{1}{\lambda_i + \lambda_j}\,\frac{\beta_i \beta_j}{\beta^2}\,[\lambda_j u(k,i) + \lambda_i u(k,j)] \ . \end{aligned} \tag{23}$$

The numerical values of the function $Y(t)$ in Eq. (10) and those of $P_{rossi}(t)$ in Eq. (13) were calculated by use of this equation for selected values of k_{eff}, corresponding to different sources. The sources selected were a Poisson source, a spontaneous fission source (^{252}Cf), and a hypothetical spallation source. The multiplicity $\bar{q}$ and the Diven factor D_q, related to the second moment of the corresponding sources as

$$D_q \equiv \frac{\langle q(q-1) \rangle}{\bar{q}^2} \tag{24}$$

[not to be confused with the functions D_k of Eq. (11)], are listed in Table III. The data regarding the spallation source were taken from Hilscher et al.[20,21]

Equations (10) and (13) were evaluated by use of Eq. (23) for three different subcriticalities and for the three different source types mentioned earlier. Results are shown in Figs. 1 and 2. The values with $k_{eff} = 0.99$ and $k_{eff} = 0.8$ were taken as two extremes. The value of $k_{eff} = 0.95$ is the most likely value to be used in a future ADS.

Regarding the Feynman-alpha values, it is seen that except for the case of the smallest subcriticality, the $Y(t)$ values differ quite significantly between the traditional and spallation sources. Differences can be observed even between the traditional and the Cf source, although those differences are not so significant. The larger the source multiplicity, the larger the amplitude of the $Y(t)$ value becomes. As noted in earlier works, this is beneficial for the application of the Feynman-alpha method in ADS.

Qualitatively similar conclusions can be drawn for the Rossi-alpha values. The amplitude of the P_{rossi} values is always larger for the spallation source than that of the Cf or Poisson sources, and the difference increases with increasing subcriticality, just as for the Feynman-alpha values.

Another aspect, investigated in the traditional cases (see, for example, Ref. 3), is the relative significance of the various terms D_i in Eq. (10). We shall investigate only the relative significance of the term D_0 as compared to the contribution with the other terms. More precisely, we shall use the so-called prompt-variance approximation,[5] i.e., assume

$$D_0 = \frac{1}{2\alpha_0^2}\left[\langle \nu_p(\nu_p - 1) \rangle + \langle q(q-1) \rangle \frac{\nu}{\bar{q}}(-\rho) \right] \ ,$$

$$D_k = 0,\ k = 1,\ldots,6 \ . \tag{25}$$

Yet another aspect to investigate is to compare the full treatment with six delayed-neutron groups with the

TABLE III

Nuclear Properties of Various Sources

	Poisson	^{252}Cf	Spallation
Yield $\bar{q}$	1	3.784	41
Diven factor D_q	0	0.8479	0.98

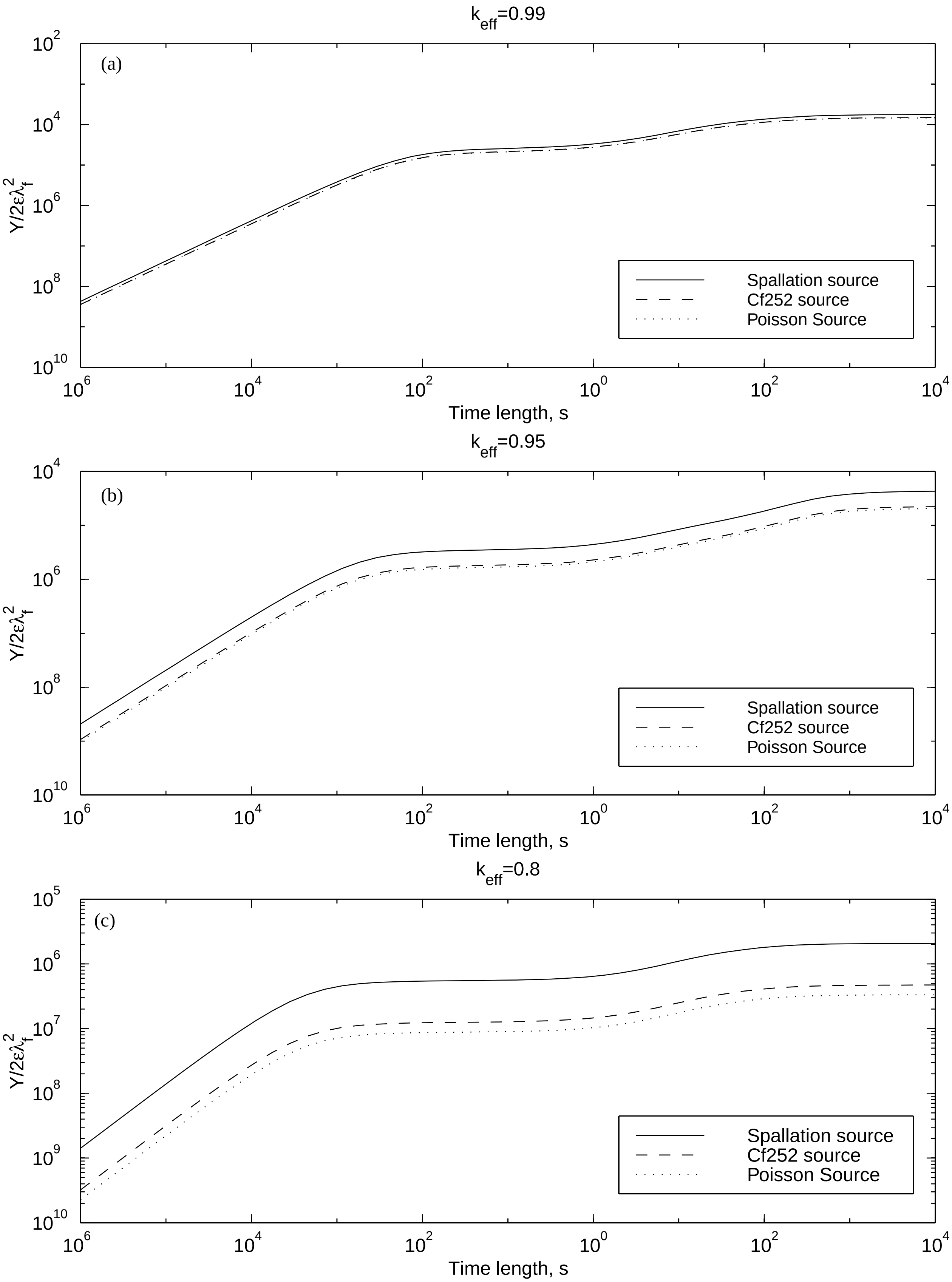

Fig. 1. Effect of different sources on the Feynman-alpha values.

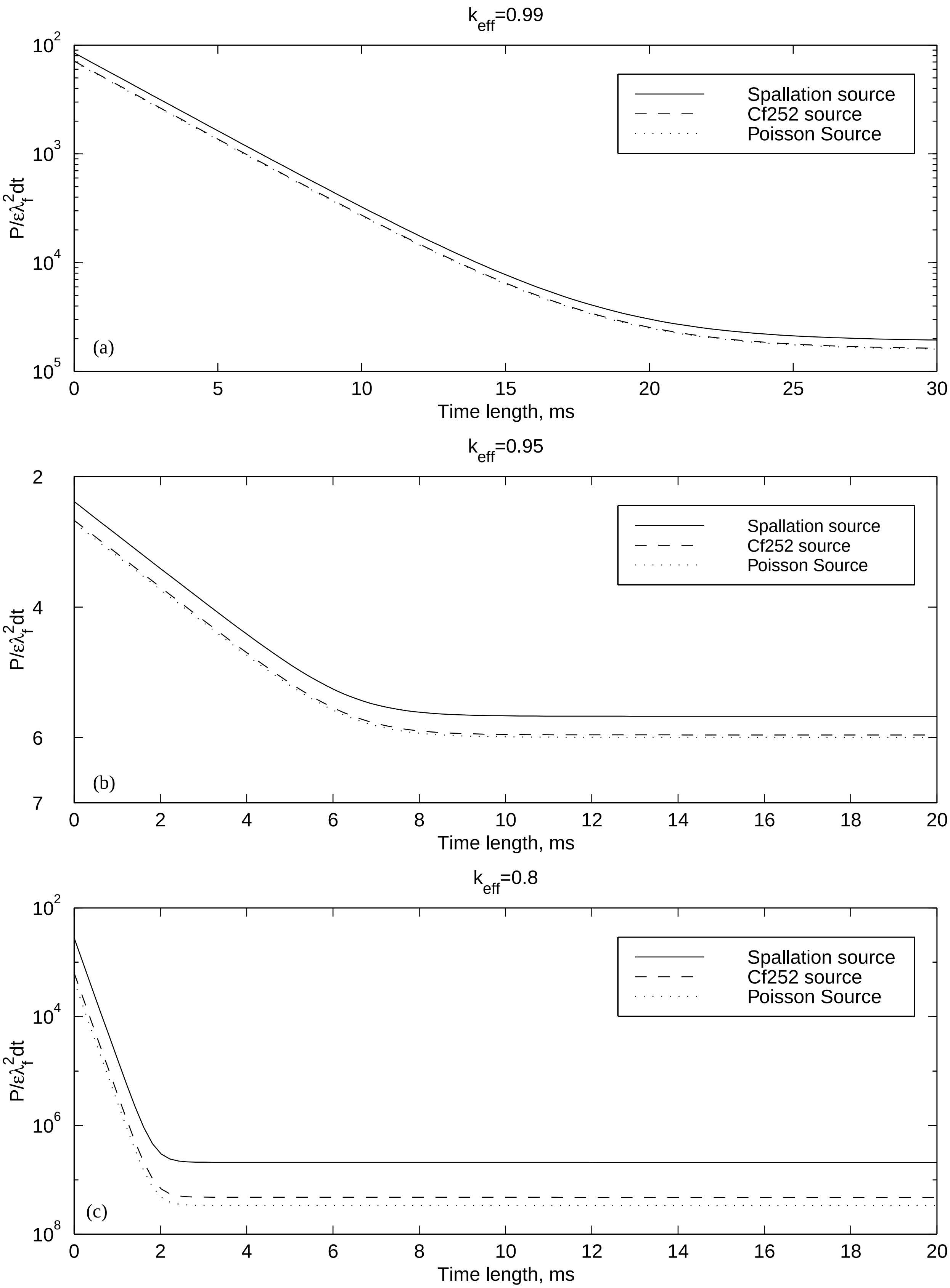

Fig. 2. Effect of different sources on the Rossi-alpha values.

simplifying approximation that there is only one single, averaged group of delayed neutrons with the parameters $\beta = 0.0065$, $\lambda = 0.077\ s^{-1}$ (Ref. 18). In this case, after some algebra, one obtains

$$Y(t) = 2\epsilon\lambda_f^2 \sum_{k=0}^{1} D_k f_k(t) \tag{26}$$

and

$$D_k = [D_\nu[\nu(1-\beta)]^2 + D_q \nu\bar{q}(-\rho)]\omega_k - [2\nu(1-\beta)\beta + (\beta-1)\beta]u(k,i) \ , \quad i \neq k \ , \tag{27}$$

where D_ν and D_q are the fission and spallation Diven factors, respectively, as defined in Eq. (24). Numerical results for two subcriticalities are shown in Fig. 3 for the Feynman method. The continuous line corresponds to the case with six delayed-neutron groups; the dashed curves correspond to the single (average) delayed-neutron group, Eqs. (26) and (27); whereas the dotted line corresponds to the prompt variance approximation, Eq. (25). It is seen that the approximation of one averaged delayed-neutron group reproduces the effect of the delayed neutrons quite poorly. On the other hand, both approximations, including the prompt variance approximation, are suitable to describe the first part of the function $Y(t)$ for short times (at least for larger subcriticalities) from which the prompt-neutron time constant is determined from experiments, at least for cases not too close to critical.

As for the Rossi-alpha method, again similar effects can be seen, as shown in Fig. 4. Namely, the effect of the various forms of the functions D_i appears at larger time lags.

V. CORRELATIONS

In the earlier sections, we discussed the Feynman- and Rossi-alpha values from the following points of view:

1. various subcritical systems with different types of extraneous sources
2. various subcritical systems without taking delayed neutrons into consideration
3. various subcritical systems with one group of delayed neutrons
4. various subcritical systems with six groups of delayed neutrons.

A reader may readily ask how significant the correlation terms of the prompt-delayed and delayed-delayed fission terms are for a subcritical system. As mentioned earlier, these terms were not calculated explicitly in earlier works.

From Eq. (23), it is seen that these correlations can be expressed by

$$Cor(t) = 2\epsilon\lambda_f^2 \sum_{k=0}^{6} Cor_k f_k(t) \ , \tag{28}$$

where

$$Cor_k = -\sum_{j=1}^{6} (2\langle \nu_p \nu_{d_j}\rangle + \langle \nu_{d_j}(\nu_{d_j}-1)\rangle) u(k,j) - 2\sum_{\substack{i,j=1 \\ i<j}}^{6} \frac{1}{\lambda_i + \lambda_j} \langle \nu_{d_i}\nu_{d_j}\rangle [\lambda_j u(k,i) + \lambda_i u(k,j)] \ . \tag{29}$$

This shows, as expected, that these terms are independent of the type of extraneous sources, since the sources do not produce delayed neutrons. Neglecting these correlations means neglecting the terms in Eqs. (28) and (29) in the full expression for $Y(t)$.

Figure 5 shows the time dependence and the magnitude of the prompt-delayed and delayed-delayed correlations, calculated by Eqs. (28) and (29), for three different reactivities. The characteristics of the time dependence, with a saturation effect at large times, are self-explaining. The relative contribution of these correlations to the total solution, i.e., $Cor(t)/Y(t)$ values for the three source types, are shown in Fig. 6 for the case of $k_{eff} = 0.95$. This relative contribution is equal to the relative error of neglecting these correlation terms in $Y(t)$. As follows from the foregoing discussion, the largest relative contribution (error) is obtained with the traditional source. The error increases with time because the prompt correlations die out faster than the prompt-delayed and delayed-delayed correlations. Nevertheless, the error is quite small in all cases, especially for short time intervals. When determining the prompt neutron time constant, only data corresponding to short times are used. For $k_{eff} = 0.95$, corresponding to Fig. 6, it is sufficient to consider data up to $t = 10^{-2}$ s (compare Fig. 1b). As Fig. 6 shows, the maximum relative error is $\sim 10^{-5}$ for such times.

This extremely small value of the relative error shows that the neglect of the delayed correlations is indeed fully justified when evaluating prompt neutron time constants from Feynman-alpha experiments. Similar conclusions can be drawn also for the Rossi-alpha method.

VI. EFFECTS OF THE REACTOR PROPERTIES

Although the effects of a multiple source, such as a spallation one, on the Feynman- and Rossi-alpha values are in principle interesting also for traditional systems, a spallation source will be primarily used in subcritical reactors having a composition different from traditional thermal systems. It is therefore interesting to perform the same

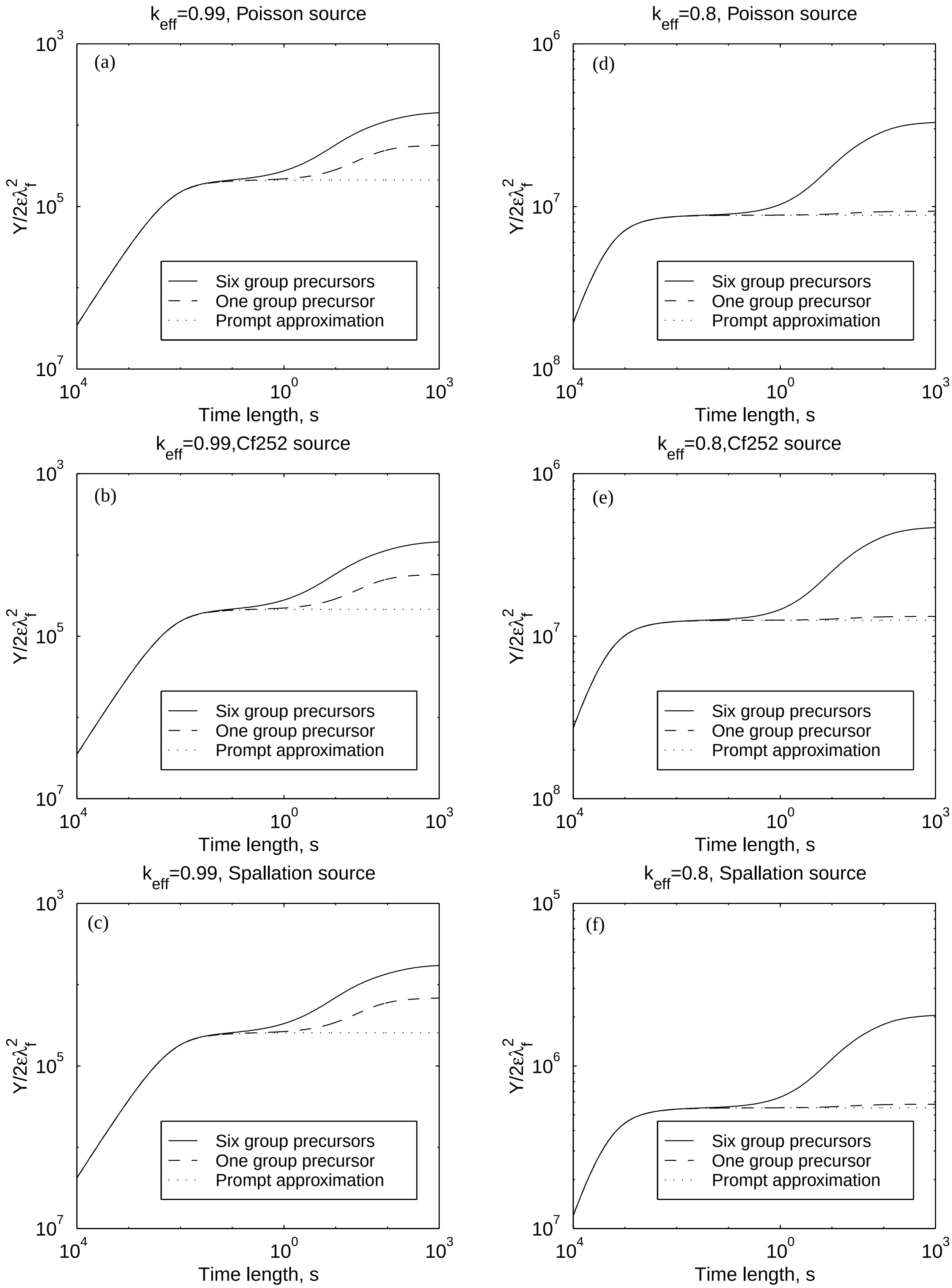

Fig. 3. Effect of various forms of the functions D_i on the Feynman-alpha values.

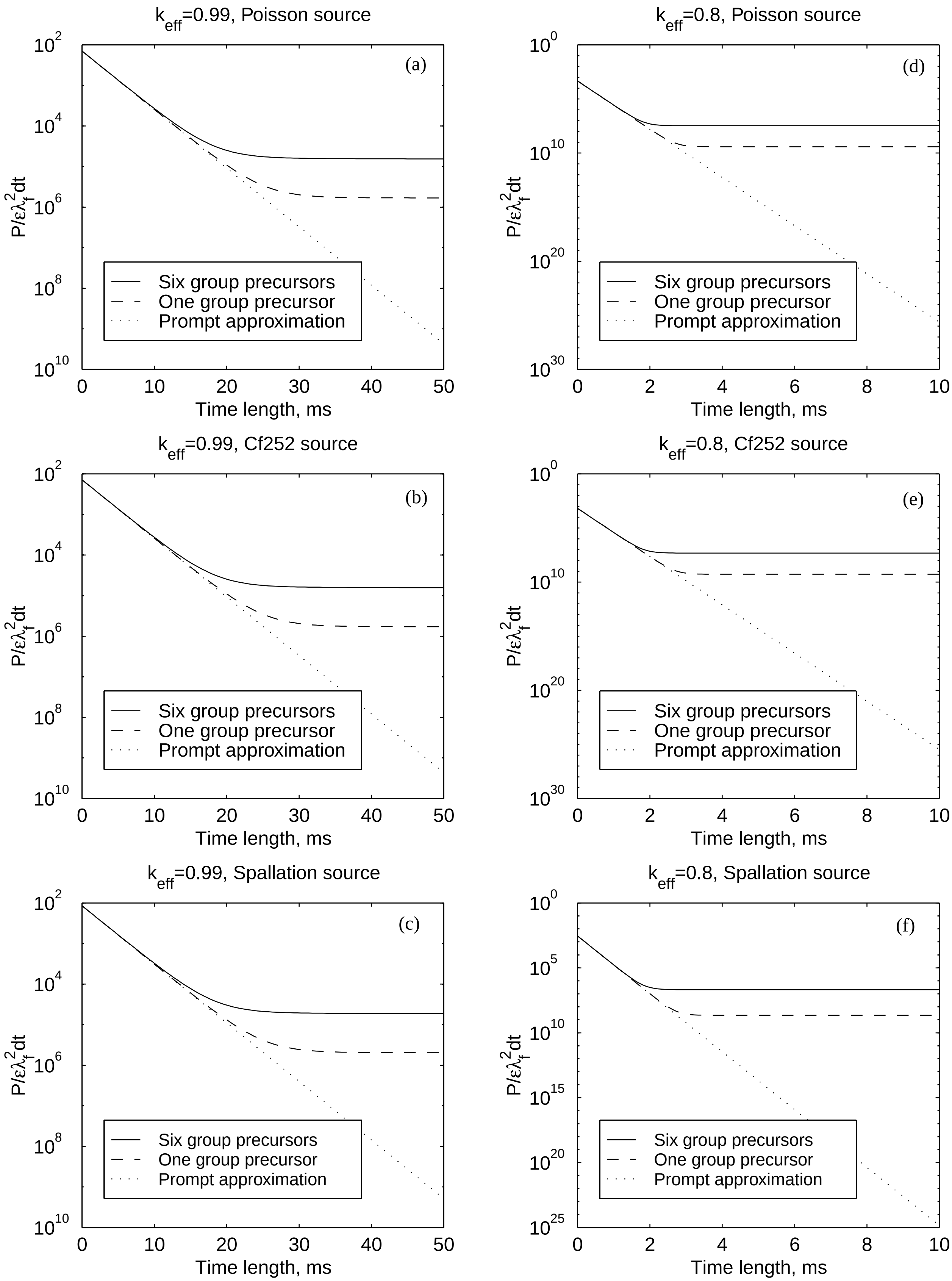

Fig. 4. Effect of various forms of the functions D_i on the Rossi-alpha values.

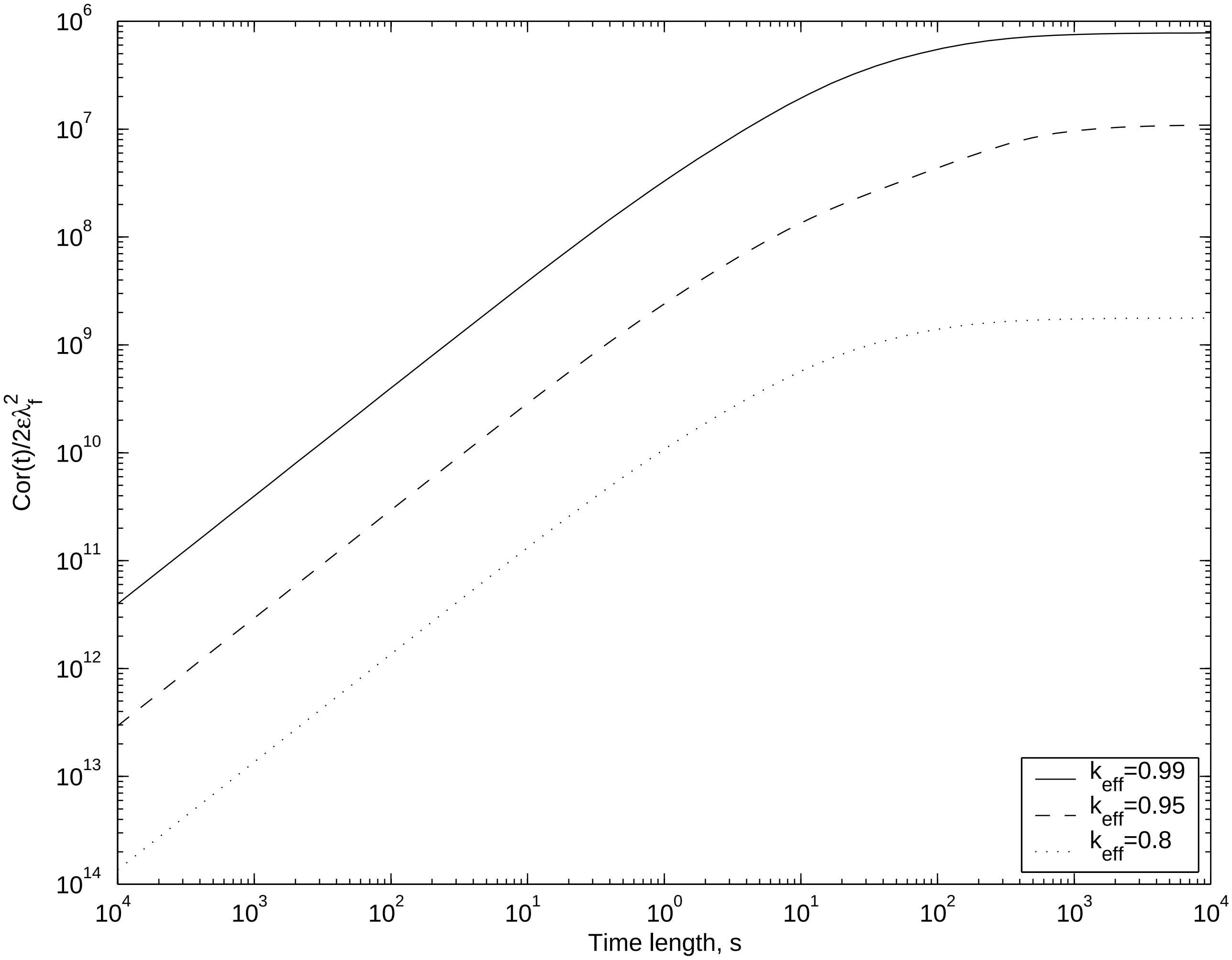

Fig. 5. Correlations of prompt-delayed and delayed-delayed neutrons for three different reactivities.

analysis with the material constants corresponding to a system that can be considered as characteristic for a future ADS core. The system is a lead-cooled one and uses ^{233}U as the fissile material and ^{232}Th as fertile material. The nuclear parameters of the system are shown in Table IV, and the roots of the characteristic equation are shown in Table V. These data were taken from Refs. 22 and 23. A comparison with the data for a traditional system, Tables I and II, show that the delayed neutron fraction in the prototype ADS system is smaller and the prompt neutron time constant much larger than in the traditional system. Not surprisingly, the prototype ADS is a much faster system, and the role of the delayed neutrons is smaller than in a traditional system.

TABLE IV

Characteristics of Delayed Groups for Fission of ^{233}U; $\beta = 0.00356$; $\nu = 2.6$; $\langle \nu(\nu - 1) \rangle = 5.408$

	Delayed Group					
	1	2	3	4	5	6
Decay constant λ_i (s^{-1})	0.0129	0.0313	0.1346	0.3443	1.3764	3.7425
Relative abundance β_i/β	0.0246	0.2087	0.1871	0.3619	0.1658	0.0519

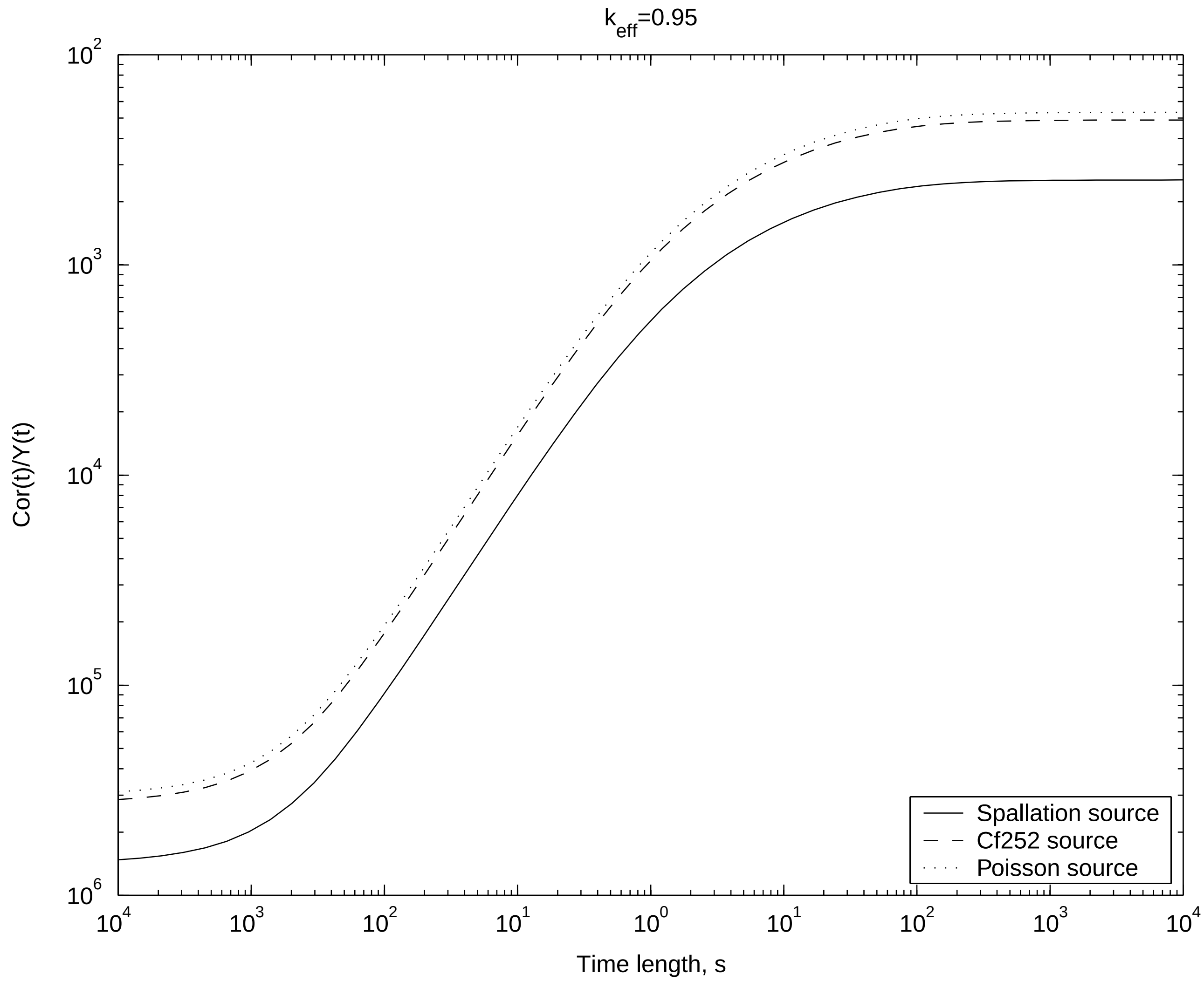

Fig. 6. The relative errors of neglect of correlations of prompt-delayed and delayed-delayed neutrons.

We shall confine here the quantitative analysis to the Feynman-alpha values only. The same quantities, and for the same cases of sources and subcriticality as shown in Figs. 1 and 3, are recalculated for a hypothetical system corresponding to a future ADS. The results are shown in Figs. 7 and 8. It is seen that the differences in the results concerning the various sources and subcriticalities remain the same as in the foregoing. However, the effect of the delayed neutrons is much smaller and appears at a later stage compared to the prompt part of the

TABLE V

Roots of the Characteristic Equations for $\Lambda = 4.2 \times 10^{-7}$ s*

k_{eff}	α	$-s_0$	$-s_1$	$-s_2$	$-s_3$	$-s_4$	$-s_5$	$-s_6$
0.99	32 537	32 537	0.00965	0.02149	0.12772	0.22448	1.34311	3.59227
0.95	133 800	133 800	0.01287	0.02896	0.13296	0.31724	0.85629	3.73322
0.8	603 725	603 725	0.01287	0.02206	0.12166	0.34099	1.36024	3.70549

*The roots $a_i = -s_i$ are given in units of s^{-1}.

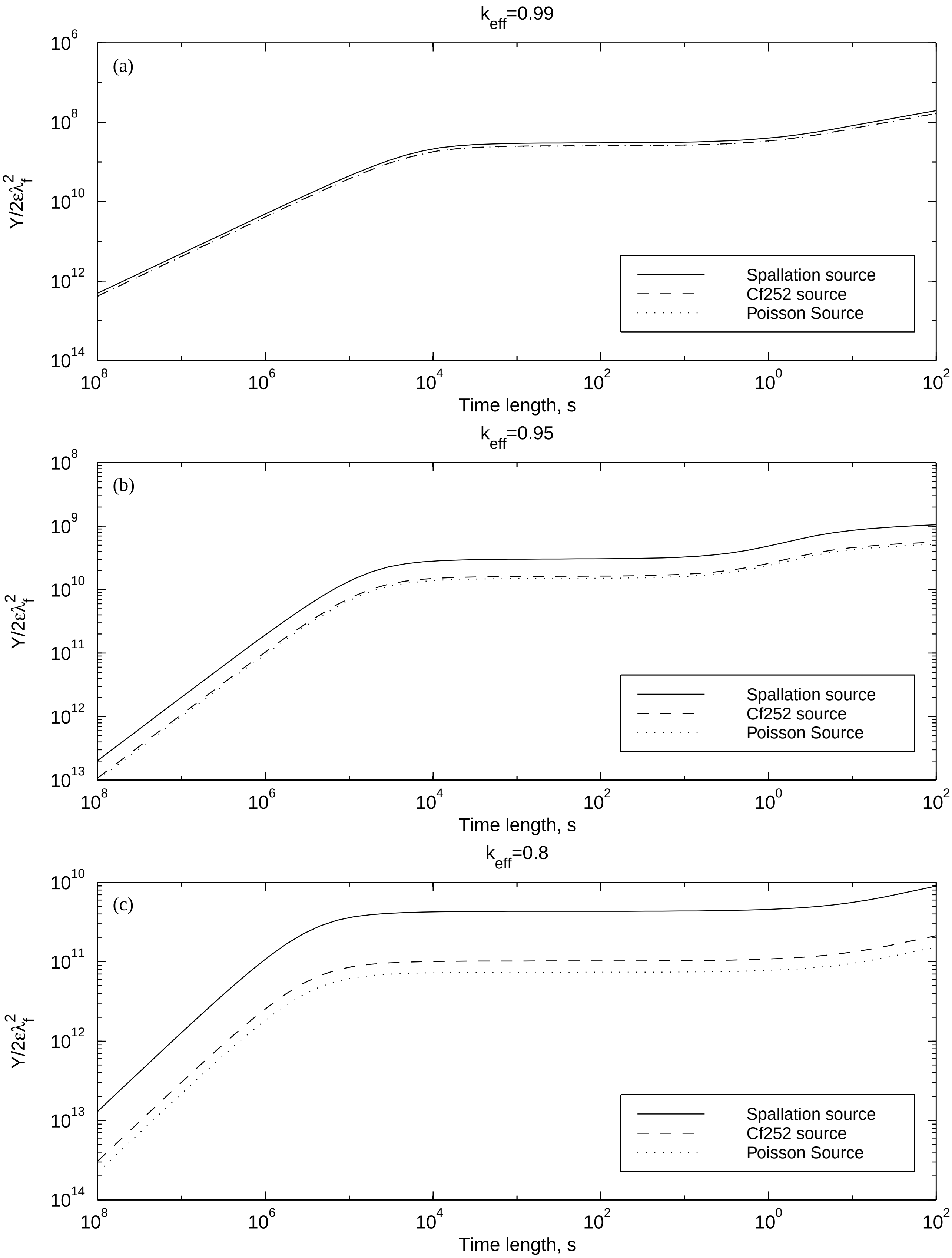

Fig. 7. Effect of different sources on the Feynman-alpha values for a prototype ADS system.

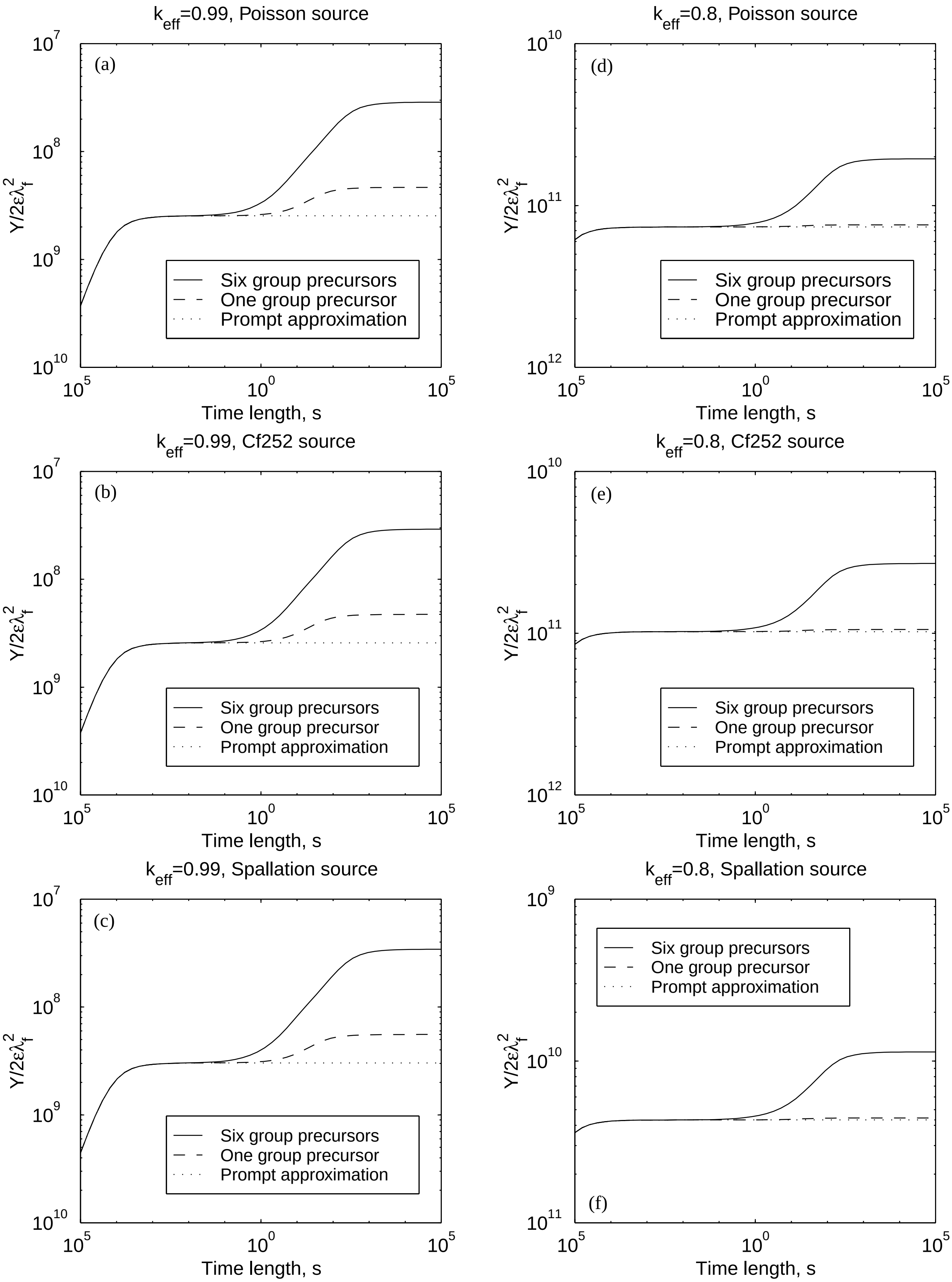

Fig. 8. Effect of various forms of the functions D_i on the Feynman-alpha values for a prototype ADS system.

curve. This underlines further the fact that in the evaluation of the measurements, when the only task is to determine the prompt neutron time constant, it is sufficient to use formulas for both the Feynman- and Rossi-alpha values that are based on one single delayed-neutron group or on the prompt-variance approximation. Another point is, as seen by the comparison of Figs. 1c and 7c, that extraction of the prompt neutron time constant requires much better time resolution for the ADS system than for the traditional one due to the much shorter prompt neutron generation time Λ.

VII. CONCLUSIONS AND FUTURE WORK

The quantitative analysis of the Feynman- and Rossi-alpha formulas for future ADS systems shows that they are in principle suitable for monitoring reactivity in such systems. The amplitude of the variance-to-mean and the correlation function is enhanced at large source multiplicity and deep subcriticalities as compared to similar values in a traditional system with a Poisson source. This circumstance may compensate for the negative effects of worsening of the statistics when performing the measurement at full power. Such measurements are not possible in traditional systems due to the overlapping of the prompt chains close to criticality.

The basic drawback of the formulas derived and evaluated so far is that, similar to the case of traditional systems, they are based on one-group (i.e., energy-independent) theory. This is a fairly applicable approximation for traditional thermal systems where the overwhelming majority of the fissions are thermal fissions of ^{235}U, and the energy span of the neutrons is not very large. In a future ADS system, due to the much higher energy of the source neutrons and hence the much larger contribution of fast and epithermal fissions, one-group theory is most likely not applicable. Application of statistical methods such as the Feynman- and Rossi-alpha methods for reactivity monitoring requires the study of a second moment of the neutron distribution in energy-dependent cases. This will be possible only with Monte Carlo techniques, and work has already started in this direction. In addition, more basic research will be needed in the source properties. As was pointed out recently,[24] a strong dependence of the energy spectrum of spallation neutrons on their number (i.e., the existence of a number-dependent neutron spectrum) introduces energy correlations between source neutrons. These need to be considered in the energy-dependent studies of the second moment of the neutron distribution, whether it is calculated analytically or with Monte Carlo. Such source energy correlations are not known at the moment; thus, they need to be determined in order to obtain correct Feynman- or Rossi-alpha formulas.

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